ON JAN 1, 2008, JOE DEPOSITS \$2,000 INTO A RETIREMENT ACCOUNT

HIS FUNDS GROW IN THE ACCOUNT BY 5% PER YEAR

IN ADDITION, ON EVERY JAN 1 AFTER THAT, HE DEPOSITS ANOTHER \$2,000.

HOW MUCH IS HIS ACCOUNT WORTH ON JAN 1, 2048?

a_n = AMOUNT IN THE ACCOUNT ON JAN 1 OF THE n-TH YEAR

a_1 = AMOUNT ON JAN 1 OF YEAR 1 (2008) = \$2000

a_2 = " " " 2 (2009)

$$= \underbrace{2000 + 2000 * 0.05}_{\text{PREVIOUS FUNDS + GROWTH}} + \underbrace{2000}_{\text{NEW DEPOSIT}}$$

$$= 2000(1+0.05)$$

$$= 2000\underbrace{(1.05)^1}_{100\% + 5\% = 105\%} + 2000$$

WANT a_{41} = AMOUNT ON JAN 1, 2048

2048 − 2008 + 1 = 41

$$a_3 = [2000(1.05) + 2000](1.05) + 2000$$

$$= 2000(1.05)^2 + 2000(1.05) + 2000$$

← GEOMETRIC SERIES (WRITTEN IN REVERSE)

$a_1 = 2000$

$r = 1.05$

$$a_n = \underbrace{2000(1.05)^{n-1} + 2000(1.05)^{n-2} + \cdots + 2000}_{n \text{ TERMS}}$$

(FINITE) GEOMETRIC SERIES $= \dfrac{a_1(r^n-1)}{r-1} = \dfrac{2000(1.05^{41}-1)}{1.05-1}$

$= \dfrac{2000(1.05^{41}-1)}{0.05} \cdot \dfrac{20}{20} = 40{,}000\,(1.05^{41}-1)$

$\uparrow \frac{1}{20}$ $\quad \approx \$255{,}679.53$

SANITY CHECK

$a_{41} > 2000 * 41$

$a_{41} > 82{,}000$

MULTIPLIER EFFECT

IF YOU SHOP AT A LOCAL BUSINESS, 70% OF WHAT YOU SPEND THERE IS THEN SPENT @ ANOTHER LOCAL BUSINESS

↑ LOCALLY OWNED + OPERATED

IF YOU BUY A TV AT A LOCAL BUSINESS FOR \$1500
THEN THE TOTAL ECONOMIC ACTIVITY DUE TO THIS PURCHASE IS

$$1500 + 1500(0.7) + 1500(0.7)^2 + 1500(0.7)^3 + \cdots$$

$*0.7 \qquad *0.7 \qquad *0.7$

INFINITE GEOMETRIC SERIES

$a_1 = 1500, \quad r = 0.7$

$|r| = 0.7 < 1$

$$\frac{a_1}{1-r} = \frac{1500}{1-0.7} = \frac{1500}{0.3}\cdot\frac{10}{10} = \frac{15000}{3} = 5{,}000$$

9.5 BINOMIAL COEFFICIENTS + BINOMIAL THEOREM

$$(x+y)^0 = 1$$
$$(x+y)^1 = x+y$$
$$(x+y)^2 = x^2+2xy+y^2$$
$$(x+y)^3 = (x+y)(x+y)^2 = (x+y)(x^2+2xy+y^2)$$
$$= x^3+2x^2y+xy^2$$
$$x^2y+2xy^2+y^3$$
$$= x^3+3x^2y+3xy^2+y^3$$

COEF'S (PASCAL'S TRIANGLE)

```
          1
         1 1
        1 2 1
       1 3 3 1
      1 4 6 4 1
    1 5 10 10 5 1
  1 6 15 20 15 6 1
```

LEFTMOST COEF = 1

RIGHTMOST COEF = 1

NOTE: 1ST TERM IN EXPANSION IS ALWAYS THE 1ST TERM IN BINOMIAL RAISED TO SAME POWER (IE. x^n) IN $(x+y)^n$

+ POWERS OF x GO DOWN BY 1
+ POWERS OF y GO UP BY 1

} FOR EACH SUCCESSIVE TERM

TERMS IN EXPANSION $= n+1$

$$(x+y)^4 = x^4+4x^3y+6x^2y^2+4xy^3+y^4$$

$$(x+y)^6 = x^6+6x^5y+15x^4y^2+20x^3y^3+15x^2y^4+6xy^5+y^6$$

EXPAND $(2x-y)^5 = (\mathbf{2x}+(\mathbf{-y}))^5$

LET $x = \mathbf{2x}$, $y = \mathbf{-y}$

$$= (2x)^5 + 5(2x)^4(-y) + 10(2x)^3(-y)^2 + 10(2x)^2(-y)^3 + 5(2x)(-y)^4 + (-y)^5$$

$$= 32x^5 + 5(16x^4)(-y) + 10(8x^3)(y^2) + 10(4x^2)(-y^3) + 5(2x)(y^4) + (-y^5)$$

$$= 32x^5 - 80x^4y + 80x^3y^2 - 40x^2y^3 + 10xy^4 - y^5$$

NOTE: ALTERNATING SIGNS +, -

EXPAND $\left(\sqrt{x} - \frac{2}{x}\right)^4 = \left(\mathbf{x^{\frac{1}{2}}} + (\mathbf{-2x^{-1}})\right)^4$

$$= (x^{\frac{1}{2}})^4 + 4(x^{\frac{1}{2}})^3(-2x^{-1}) + 6(x^{\frac{1}{2}})^2(-2x^{-1})^2 + 4(x^{\frac{1}{2}})(-2x^{-1})^3 + (-2x^{-1})^4$$

$=$